Math 55 Discussion problems 26 Jan

1. Use a direct proof to show that the product of two rational numbers is rational.

2. Prove that if n is an integer and $3n + 2$ is even, then n is even using

   (a) a proof by contraposition.

   (b) a proof by contradiction.

3. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

4. Show that these three statements are equivalent, where a and b are real numbers: (i) a is less than b, (ii) the average of a and b is greater than a, and (iii) the average of a and b is less than b.

5. Prove that there are no positive perfect cubes less than 1000 that are the sum of the cubes of two positive integers.

6. Prove that given a nonnegative integer n, there is a unique nonnegative integer m such that $m^2 \leq n < (m+1)^2$.